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Higher excitations of the D and D_s mesons

A. M. Badalian*

Institute of Theoretical and Experimental Physics, Moscow, Russia

B. L. G. Bakker†

Department of Physics and Astronomy, Vrije Universiteit, Amsterdam, The Netherlands

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The masses of higher $D(nL)$ and $D_s(nL)$ excitations are shown to decrease due to the string contribution, originating from the rotation of the QCD string itself: it lowers the masses by 45 MeV for $L = 2$ ($n = 1$) and by 65 MeV for $L = 3$ ($n = 1$). An additional decrease ~ 100 MeV takes place if the current mass of the light (strange) quark is used in a relativistic model. For $D_s(1^3D_3)$ and $D_s(2P_1^H)$ the calculated masses agree with the experimental values for $D_s(2860)$ and $D_s(3040)$, and the masses of $D(2^1S_0)$, $D(2^3S_1)$, $D(1^3D_3)$, and $D(1D_2)$ are in agreement with the new *BABAR* data. For the yet undiscovered resonances we predict the masses $M(D(2^3P_2)) = 2965$ MeV, $M(D(2^3P_0)) = 2880$ MeV, $M(D(1^3F_4)) = 3030$ MeV, and $M(D_s(1^3F_2)) = 3090$ MeV. We show that for $L = 2, 3$ the states with $j_q = l + 1/2$ and $j_q = l - 1/2$ ($J = l$) are almost completely unmixed ($\phi \approx -1^\circ$), which implies that the mixing angles θ between the states with $S = 1$ and $S = 0$ ($J = L$) are $\theta \approx 40^\circ$ for $L = 2$ and $\approx 42^\circ$ for $L = 3$.

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I. INTRODUCTION

Till recently only the low-lying $1S$ and $1P_J$ states of the D and D_s mesons were known from experiment [1]. The situation has changed recently owing to discoveries of new D_s resonances: $D_s(2710)$ [2,3], $D_s(2860)$ [2,4], and $D_s(3040)$ [4]. Also last year, new $D(L)$ states were observed by the *BABAR* Collaboration [5]: $D(2550)$, $D^*(2600)$, $D_J(2750)$, and $D_J^*(2760)$, and in [6] the mass of $D(1^3P_0)$ was measured with a good accuracy.

The quantum numbers and decay modes of the new resonances were intensely discussed in a large number of recent studies [7–14] and also before in [15–25]. For $D_{s1}^*(2710)$ the quantum numbers $J^P = 1^-$ were assigned [4,7], although the analysis in Ref. [8] does not exclude that $D_{s1}^*(2710)$ is an admixture of $D_s(2^3S_1)$ and $D_s(1^3D_1)$. The relatively narrow resonance $D_{sJ}(2860)$ with $\Gamma = 48 \pm 3(\text{stat})$ MeV is mostly considered as the $L = 2$ state with $J^P = 3^-$ [8–10], while in Ref. [25] the resonance with close values of the mass and width has the quantum numbers $J^P = 0^+$. The wide resonance $D_{sJ}(3040)$ is mostly assumed to be the $2P$ state with $J^P = 1^+$ [11].

The quantum numbers of the new D^0 resonances and their isotopic partners were discussed in [12–14], where the broad resonance $D(2550)$ ($\Gamma \sim 130$ MeV) is considered as the singlet 2^1S_0 state, for which a large width as in experiment was obtained in Ref. [12], while a much smaller total width was calculated in [13]. The resonance $D^*(2600)$ with $\Gamma = 93 \pm 6 \pm 13$ MeV is consistent with the excited 2^3S_1

state [14], or an admixture of the 2^3S_1 and 1^3D_1 states with large mixing angle [12,13]. The resonances $D_J(2760)$ and $D_J(2750)$ have relatively small widths, $\Gamma \sim 60$ – 70 MeV, and for them the quantum numbers $J^P = 3^-$ and $J^P = 2^-$, respectively, were assigned in Refs. [13,14], while in [19] (the second paper) these two resonances are considered as the same 1^3D_1 state with $J^P = 1^-$.

These new data are extremely important for the theory to better understand the $q\bar{Q}$ dynamics and test predictions made in a large variety of models [15–25], some of which were made long ago [15,16]. For low-lying states, the theoretical predictions are mostly in agreement with experiment within 20–50 MeV accuracy, although the parameters used may be very different. This is not surprising, because the very masses of low-lying states are usually used as a fit to determine the quark masses and parameters of the potentials. On the contrary, for higher states different predictions for the masses and the fine-structure (FS) splittings were obtained in different models.

The new experimental data on the hyperfine (HF) splittings show that their values, ≈ 70 MeV, coincide for $D_s^*(2710)$ and $D_s(2638)$ [26], $D^*(2610)$ and $D(2540)$. The latter HF splitting was predicted in Ref. [27], if a “universal” coupling $\alpha_{\text{HF}} = 0.31$ is used in the HF potential.

Also the experimental mass differences between $D_s^*(2710)$ and $D^*(2610)$, $D_s(2638)$ and $D(2540)$, $D_s(2860)$ and $D(2760)$ appear to be ≈ 100 MeV, the only exception being the $D_s(1P)$ multiplet, where, on the one hand, the masses of $D_{s2}^*(2573)$ and $D_2^*(2460)$, $D_{s1}(2535)$ and $D_1(2422)$ also differ by ~ 110 MeV, while $M(D_{s0}^*(2317)) - M(D_0(2300)) \approx 25$ MeV and $M(D_{s1}(2460)) - M(D_1(2430)) \approx 30$ MeV

*badalian@itep.ru

†b.l.g.bakker@vu.nl

are much smaller. Such small FS splittings cannot be explained within the universal description of fine structure used in Refs. [15,16,18].

This discrepancy has stimulated a lot of studies to understand why D_{s0}^* and D_{s1} have such small widths, $\Gamma < 3.8$ MeV [1], and large mass shifts. It has become understood that the dynamics of the $D_{sJ}(1P)$ multiplet is different for the states with the total angular momentum of the s quark $j_s = 1/2$ and $j_s = 3/2$ [21–25] and the bispinor structure of the $D_s(1P)$ wave function (w.f.) and the w.f. of the $D(1S)$ mesons in the decay channel are very important. Two factors provide a large hadronic shift: the nearby S -wave threshold and the large overlap integral between the upper components of the $D_{sJ}(1P)$ w.f. with $j_s = 1/2$ and the lower components of the w.f. of the D meson in the decay channel [23].

Surprisingly, there are no large mass shifts for the other excited D and D_s states observed, and as a whole, the single-channel description turns out to be a useful tool to understand the general structure of the D and D_s spectra and FS splittings, and to predict the masses of the yet undiscovered resonances. Till now one of the best predictions for the meson masses of the low-lying states was obtained in the QCD motivated relativistic quark model (RQM), back in 1985 [15].

However, within a single-channel approximation such important characteristics as partial widths and possible mass shifts of the resonances cannot be defined and our calculations may be considered as the first step to understand the dynamics of excited D and D_s mesons. To calculate partial widths, one needs to consider both chiral decay modes like $D(nL) \rightarrow D^*\pi$ or $D_s(nL) \rightarrow D^*K$, etc., when chiral effects may be of special importance as in Ref. [23], and also strong decays like $D \rightarrow D^*\rho$ which may originate from the string breaking. Some strong decay mechanisms were suggested and discussed in Refs. [7–14].

In contrast to the low-lying states, discrepancies show up for higher states, which may reach ~ 100 MeV. For example, for $D_s(3^1S_0)$ the masses 3097 MeV from the paper Ref. [7] and 3259 MeV [16], and for $M(D_s(2^3P_2))$ the values 3041 MeV [7] and 3157 MeV [16,17], were obtained, showing differences ≥ 100 MeV. The reasons why they occur will be discussed in the present paper.

A comparison of the results obtained in different models is simplified, if the same value of the string tension σ is used. The choice of σ is of great importance, because the meson mass is proportional to $\sqrt{\sigma}$ for the linear potential σr , which dominates for higher states. However, much different values of σ are used in potential models: a large $\sigma \sim 0.26$ GeV² in [16,17], small $\sigma = 0.115$ GeV² in [24], and $\sigma = 0.14$ GeV² in [20]. Here we use $\sigma = 0.18$ GeV², which follows from the analysis of the Regge trajectories for light mesons, and was already used in Refs. [15,19]. Taking the same σ , one can establish common features and differences between the relativistic string Hamiltonian

(RSH) [28] used here and the RQM developed in Refs. [15,19]. In particular, we show that the choice of the current light (strange) quark mass is of special importance in relativistic models.

The only uncertainty in our calculations comes from the gluon-exchange (GE) potential, since at present there is no consensus about the value of the vector coupling at large distances, called the freezing constant or the critical constant α_{crit} . In Ref. [15] the value $\alpha_{\text{crit}} = 0.60$ was used and the variation of α_{crit} in the range 0.60 ± 0.10 produces rather small changes, ≤ 20 MeV, in the spin-averaged masses for higher states. However, the value of the strong coupling in spin-orbit and tensor potentials, $\alpha_{\text{FS}}(\mu)$, is also not fixed now, in contrast to the FS in heavy quarkonia, where the scale μ and second order perturbative corrections are known [29], giving $\alpha_{\text{FS}}(\mu)$ smaller than α_{crit} [30]. In our analysis of the FS here, we shall test different values of α_{FS} to fit new experimental data on the $D(1D)$ multiplet.

In our paper we concentrate on the multiplets with $L = 2, 3$; to calculate mixing angles for states with $J = L$ we use the basis \mathbf{j}_q^2 from Ref. [22], where $\mathbf{j}_q = \mathbf{l} + \mathbf{s}_q$ is the total angular momentum of the light (strange) quark and the total spin $\mathbf{J} = \mathbf{j}_q + \mathbf{s}_Q$ is the sum of the light-quark total angular momentum and the spin of the heavy quark. It appears that for higher states with $J = l$ the states with $j_q = l + 1/2$ and $j_q = l - 1/2$ are in fact unmixed, $|\phi(nl)| \approx 1^\circ$, and this result may be important to study different decay modes of heavy-light mesons. Owing to the known relation between the mixing angle $\phi(nl)$ and the mixing angle $\theta(nL)$ in the S^2 scheme (or \mathbf{LS} scheme with $\mathbf{J} = \mathbf{L} + \mathbf{S}$), the states with the spin $S = 1$ and $S = 0$ ($J = L$) appear to be mixed with large mixing angle, e.g., $\theta(1D) = 40.2^\circ$.

II. RELATIVISTIC STRING CORRECTIONS

Here we use the RSH, derived for spinless quarks and antiquarks [28], while all spin-dependent interactions are considered as a perturbation. To calculate the spectra of the heavy-light mesons this approach has some advantages as compared to the use of the Dirac equation (DE) and considering the heavy-quark contribution as $1/m_Q$ corrections [16,17]. As shown in Ref. [31], for a scalar potential the solutions of the DE have an important property: the spectrum is symmetric under the reflection of the eigenvalues (e.v.), $\epsilon_n \rightarrow -\epsilon_n$, so that negative energy states are in fact not present in the spectrum of a heavy-light meson.

Moreover, from the expression for the squared e.v. ϵ_n^2 of the DE (with a given $l = l_q$ and $j = j_q$ —the total angular momentum of a light quark) it follows that the mass difference between neighboring states is equal to $\epsilon_{n+1}^2 - \epsilon_n^2 \simeq 4\sigma_D + \ln(\frac{\epsilon_{n+1}^2}{\sigma_D|\kappa|}) - \ln(\frac{\epsilon_n^2}{\sigma_D|\kappa|})$ [31], where σ_D is the string tension used in the DE and the constant κ enters the Coulomb interaction, $-\frac{\kappa}{r}$. For the DE this mass difference (for a given σ) appears to be significantly smaller

than that in the RSH and the RQM, where it is equal to $4\pi\sigma$.

Just to compensate such a small spacing between radial excitations the larger value of the string tension, $\sigma_D \approx 0.26 \text{ GeV}^2$, is needed [16,17] (in both cases the $1/m_Q$ corrections were taken into account). However, it remains unclear why in Ref. [16] the calculated values $M(D_s(1^3P_0)) = 2487 \text{ MeV}$ and $M(D_s(1P_1^H)) = 2605 \text{ MeV}$ (for $j_q = 1/2$) are similar to the numbers obtained in the RQM [15,23], while much smaller values, 2325 MeV and 2467 MeV, were calculated within a similar approach in Ref. [17]. Here we will mostly compare our results with those models [15,19], where the same $\sigma = 0.18 \text{ GeV}^2$ was used, and draw definite conclusions about the dynamics of the $q\bar{Q}$ interaction.

The RSH $H = H_0 + H_{\text{str}}$ for spinless quarks and antiquarks was derived in instantaneous approximation [28] and has the following characteristic features:

- (1) The QCD string, besides a standard rotation of a quark and an antiquark, rotates itself, giving an additional contribution to a Hamiltonian, H_{str} . For heavy-light mesons such string corrections are not large, $\sim 30\text{--}70 \text{ MeV}$ (for $L = 1, 2, 3$), and can be considered as a perturbation [18], while in light mesons the string corrections may dominate for states with large L [32].
- (2) In the unperturbed Hamiltonian $H_0 = T + V_B$ the kinetic term T [28] is

$$T = \frac{\omega_1}{2} + \frac{m_1^2}{2\omega_1} + \frac{\omega_2}{2} + \frac{m_2^2}{2\omega_2} + \frac{\mathbf{p}^2}{2\omega_{\text{red}}}, \quad (1)$$

where by derivation the quark mass cannot be chosen arbitrarily and must be equal to the current mass \bar{m}_q for the u , d , and s quarks and the pole mass m_Q for a heavy quark, thus taking into account perturbative corrections to the heavy-quark mass. In our calculations $\bar{m}_q = 0$ for the u , d quarks, $m_s \approx \bar{m}_s(1 \text{ GeV}) = 200 \text{ MeV}$ for the s quark, and the conventional pole mass $m_c(\text{pole}) = 1.42 \text{ GeV}$ for the c quark [1] are used. This choice of m_s is similar to that in Ref. [16], where $m_s = 220 \text{ MeV}$ was used in the DE, being larger compared to $\bar{m}_s(2 \text{ GeV}) = 95 \pm 20 \text{ MeV}$ at the scale $\mu = 2 \text{ GeV}$ [1]. The reason for that difference possibly originates from the fact that in the Hamiltonian approach the s -quark current mass $\bar{m}_s(\mu)$ enters at a smaller scale, $\mu \sim 1 \text{ GeV}$ [33]. The value we take here, $m_s = 200 \text{ MeV}$, is significantly smaller than $\bar{m}_s \sim 500 \text{ MeV}$ used in constituent quark models [19,20]. It is important that the use of current quark masses allows us to avoid several fitting parameters (constituent masses).

- (3) The value of the string tension $\sigma = 0.18 \text{ GeV}^2$ cannot be used as a fitting parameter, as it is fixed

by the slope of the Regge trajectories for light mesons.

- (4) The choice of the GE potential is important for low-lying states. Here we use the vector strong coupling $\alpha_B(r)$ which possesses the asymptotic freedom (AF) property and freezes at large distances at the value α_{crit} . For higher excitations the choice of α_{crit} becomes less important; moreover, in many cases the GE potential can be considered as a perturbation.

In the RSH $H = H_0 + H_{\text{str}}$ the unperturbed part

$$H_0 = T(\omega_1, \omega_2) + V_B(r) \quad (2)$$

contains the kinetic term T (1), where the variables ω_i have to be determined from the extremum conditions: $\frac{\partial H_0}{\partial \omega_i} = 0$ ($i = 1, 2$) [28,34]. Then one finds

$$\omega_i(nL) = \left\langle \sqrt{\mathbf{p}^2 + m_i^2} \right\rangle_{nL} \quad (i = 1, 2). \quad (3)$$

The kinetic energy of a light (strange) quark is denoted as $\omega_1(nL) = \omega_q(nL)$, and $\omega_2(nL) = \omega_c$ is the kinetic energy of the c quark; the quantity $\omega_{\text{red}} = \frac{\omega_1 \omega_2}{\omega_1 + \omega_2}$, and $\mathbf{L} = \mathbf{L}_1 + \mathbf{L}_2$. Then putting ω_i into Eq. (1), one arrives at a different form of T , denoted below as T_R :

$$T_R = \sqrt{\mathbf{p}^2 + m_q^2} + \sqrt{\mathbf{p}^2 + m_c^2}. \quad (4)$$

Rigorously, the expression (4) for T_R is valid only for $L = 0$, while in general, for $L \neq 0$, $T = T_R + T_{\text{str}}$ contains the kinetic energy of the string rotation T_{str} . For $L \leq 4$ this term T_{str} is small compared to T_R and can be considered as a perturbation; its matrix element (m.e.) $\Delta_{\text{str}}(nL) = \langle T_{\text{str}} \rangle_{nL}$ is included in the mass formula (6). The form T_R of the kinetic energy was suggested in Ref. [35] and used in many models [15,36], while due to our derivation of T_R one can establish the connection between the unperturbed RSH H_0 and the RQM, where the same kinetic term is used.

Then the e.v. $M_0(nL)$ and w.f. are defined by the spinless Salpeter equation:

$$[T_R + V_B(r)]\varphi_{nL} = M_0(nL)\varphi_{nL}. \quad (5)$$

It is essential that in the RSH approach the spin-averaged meson mass $M(nL) \equiv M_{\text{cog}}(nL)$ is not only defined by the e.v. $M_0(nL)$ (5), but also contains two additional negative contributions: the string correction $\Delta_{\text{str}}(nL) = \langle H_{\text{str}} \rangle_{nL}$ [18,34] and the nonperturbative self-energy (SE) term $\Delta_{\text{SE}}(nL)$ [37]:

$$M(nL) = M_0(nL) + \Delta_{\text{str}}(nL) + \Delta_{\text{SE}}(nL). \quad (6)$$

For a given radial quantum number n , the string correction increases for larger L , while for a given L it decreases for higher radial excitations. For the $D(1P)$, $D(1D)$, and $D(1F)$ states their values are equal to $\sim -23 \text{ MeV}$, -45 MeV , and -65 MeV , respectively, which can be obtained using the analytical expressions for Δ_{str} from Ref. [18]. As an illustration, in Table I the masses

TABLE I. The D and D_s masses for the $1D$, $2P$, and $1F$ states (in MeV).

State	Ref. [15]	This paper	Exp. Refs. [2–5]
$D(1^3D_3)$	2830	2760	2762
$D_s(1^3D_3)$	2920	2840	2860
$D_s(2P_1^H)$	Absent	3040	3044
$D(1^3F_4)$	3110	3030	Absent

calculated here for several $D(nL)$ and $D_s(nL)$ states are compared to those from Ref. [15]. It appears that differences between them are mostly due to string corrections, ~ 40 MeV, and our numbers are closer to the experimental data [2–5].

For the $1P$ and $2P$ states the string corrections are smaller, -22 MeV and -10 MeV, respectively, and the masses $M(D_s(2P_1^H))$ calculated here coincide with the experimental mass of $D_s(3040)$, if this resonance with $J^P = 1^+$ is identified as the higher $2P_1^H$ state with $j_s = 1/2$ (which has to have a larger total width), while in the low-mass state with $M(2P_1^-) = 3020$ MeV the state with $j_s = 3/2$ dominates.

Even larger mass differences occur for the yet unobserved states $D(2^3P_2)$, and $D(n^3F_4)$ ($n = 1, 2$) for which the masses we predict here are ~ 100 MeV smaller than in [19] (see Tables II and III).

The perturbative self-energy correction contributes to the current mass of a heavy quark (it gives $\sim 15\%$ for a c quark [1]) and moreover there exists a nonperturbative SE correction to the quark (antiquark) mass. This correction is very important to provide the linear behavior of the Regge trajectories [34]. As shown in Ref. [37], this correction is flavor-dependent and strongly depends on the current quark mass, being small for a heavy quark and large for a light (strange) quark [37]:

$$\Delta_{\text{SE}} = -\frac{3\sigma}{2\pi} \left(\frac{\eta_f}{\omega_q(nL)} + \frac{\eta_Q}{\omega_Q(nL)} \right). \quad (7)$$

The factor η_f is determined by the quark current mass and the vacuum correlation length [37,38]: $\eta_f = 1.0$ for a light quark, $\eta_s = 0.70$ for the s quark, and $\eta_c = 0.35$ for the c quark. Notice that the number $3/2$ enters the SE term (7),

Ref. [15]	$m_{u,d} = 220$ MeV,	$m_s = 419$ MeV,	$m_c = 1628$ MeV,
Ref. [19]	$m_{u,d} = 330$ MeV,	$m_s = 500$ MeV,	$m_c = 1550$ MeV,
this paper	$m_{u,d} = 0$,	$m_s = 200$ MeV,	$m_c = 1420$ MeV.

Our results are presented in Tables II and III. The mass $M(D(1^3D_3)) = 2760$ MeV calculated here coincides with the experimental mass of $D(2760)$, which is assumed now to be the $J^P = 3^-$ state [13,14]. This value is smaller than the masses 2863 MeV given in Ref. [19] and 2830 MeV

instead of the number 2 in [37]; this change follows from a more exact definition of the vacuum correlation length [38].

From Eq. (7) one can see that the kinetic energies ω_i play a special role: they determine both the string and the SE contributions, and also enter all spin-dependent potentials [39]. In some potential models a negative overall constant C_0 is introduced, which may play the role of a self-energy correction; however, such a constant violates the linear behavior of the Regge trajectories. It is also important that in the RSH the SE terms decrease for higher states, being proportional to $\omega_q^{-1}(nL)$.

We use here the “linear + GE” static potential, $V_B(r)$, which was already tested in a number of our previous works devoted to heavy-light mesons [18,23] and heavy quarkonia [40]:

$$V_B(r) = \sigma r - \frac{4\alpha_B(r)}{3r}, \quad (8)$$

where the vector coupling $\alpha_B(r)$ is taken as in background perturbation theory [41] with $\alpha_{\text{crit}} = 0.50$, which is a bit smaller than $\alpha_{\text{crit}} = 0.60$ in Ref. [15], while a larger value $\alpha_{\text{crit}} = 0.84$ was used in Ref. [19]. In all cases the AF behavior of the vector coupling is taken into account. Notice that if a constant value α_0 (without AF behavior) is used in the GE potential, then the value of α_0 turns out $\sim 30\%$ smaller than α_{crit} .

III. HIGHER D MESONS

The masses of higher D excitations are presented in Tables II and III together with results from [15,19], where the same $\sigma = 0.18$ GeV² is used. In these tables we have omitted results for the ground states, 1^1S_0 , 1^3S_1 , and the $1P$ states, since they were studied in detail within the same approach in Ref. [18]; also for low-lying states their masses do not differ much in different models, since they are often used as a fit.

On the contrary, for higher states, a large effect takes place when the constituent quark masses, instead of the current masses, are used. In Refs. [15,19] the following masses were taken:

given in Ref. [15] and this difference is partly explained by the string correction, equal to -45 MeV.

Much larger differences occur for the excitations with $n = 2$ and $L = 2, 3$. For example, $M(2^3D_3) = 3212$ MeV is obtained here, while the value 3335 MeV was predicted

TABLE II. The D meson masses $M(nS)$ ($n = 2, 3$) and $M(nD)$ ($n = 1$) (in MeV). The experimental data are from [5]; the quark masses are given in (9); $\sigma = 0.18 \text{ GeV}^2$ in all cases.

State	Experiment Ref. [5]	This paper	Ref. [15]	Ref. [19]
2^1S_0	2539	2567	2580	2581
2^3S_1	2608	2639	2640	2632
3^1S_0	absent	3065		3062
3^3S_1	Absent	3125		3096
1^3D_1	^a	2790	2820	2788
1^3D_3	2763	2760	2830	2863
$1D_2^H$	Absent	2810		2850
$1D_2^L$	2750	2746		2806
2^3D_1	Absent	3215		3228
2^3D_3		3212		3335

^aThe identification of this state is not certain, because the quantum numbers of the $D(2760)$ state are not established. It could be either a 1^3D_3 or a 1^3D_1 state.

TABLE III. The D meson masses $M(nP)$, $M(nF)$ ($n = 1, 2$) (in MeV).

State	This paper	Ref. [15]	Ref. [19]
2^3P_0	2880		2919
$2P_1^H$	2960		3021
$2P_1^L$	2940		2932
2^3P_2	2965		3012
1^3F_4	3030	3110	3187
1^3F_2	3088		3090
2^3F_4	3430		3610

in Ref. [19], and this result cannot be explained by a string correction, which is only $\sim -25 \text{ MeV}$ in this case. From our point of view it happens due to the use of a large constituent mass for a light quark.

In Tables II and III we denote by P_1^H and D_2^H the high-mass states with $J = L$, and by P_1^L and D_2^L the low-mass states. Each of these states is an admixture of the state with $j_q = l + 1/2$ and $j_q = l - 1/2$ in the j_q^2 scheme, and for $l = 2, 3$ the mixing angle between these states appears to be small (see Sec. V).

For the 2^3P_2 state we predict the mass, 2965 MeV, smaller than the values 3012 MeV in Ref. [19] and 3035 MeV in Ref. [16]. For the states with $L = 3$ calculated here, the mass $M(D(1^3F_4)) = 3030 \text{ MeV}$ is 157 MeV and 80 MeV smaller than in Refs. [15,19], respectively, and these large differences can be only partly explained by the string correction, equal to -65 MeV for the $1F$ states. The largest difference takes place here for $M(2^3F_4) = 3430 \text{ MeV}$, which is much smaller than the value 3610 MeV from Ref. [19]. Again, such a large discrepancy cannot be explained by a string correction, which is $\sim -48 \text{ MeV}$ for the $2F$ states.

TABLE IV. The masses $M(nL)$ (in MeV) for D_s mesons.

State	Refs. [1–4]	This paper	Ref. [15]	Ref. [19]
2^1S_0	2638 ^a	2656	2670	2688
2^3S_1	2710 2688 ^b	2728	2730	2731
3^1S_0	Absent	3140		3219
3^3S_1	Absent	3200		3242
2^3P_0	Absent	2970		3054
$2P_1^H$	3044	3040		3154
$2P_1^L$	Absent	3020		3067
2^3P_2	Absent	3045		3142
1^3D_1	Absent	2870	2900	2913
$1D_2^H$	Absent	2885		2961
$1D_2^L$	Absent	2828		2931
1^3D_3	2862	2840	2920	2973
2^3D_1	Absent	3290		3383
2^3D_3	Absent	3285		3469
1^3F_4	Absent	3110	3190	3300
1^3F_2	Absent	3150		3230
2^3F_4	Absent	3490		3754

^aThe data of SELEX [26]

^bThe data of Belle [3]

In Tables I, II, III, and IV all FS splittings given are calculated taking the strong coupling α_{fs} in the spin-orbit and tensor potentials equal to 0.45.

We can summarize our results for the higher D mesons:

- (1) The HF splitting between $D(2^3S_1)$ and $D(2^1S_0)$, equal to 72 MeV, was calculated with the use of the “universal” strong coupling in the HF potential, $\alpha_{HF} = 0.31$ from Ref. [27]; this splitting is in full agreement with experiment.
- (2) The string corrections, present in the RSH, reduce the spin-averaged masses of the nL multiplets by -25 MeV , -45 MeV , -65 MeV for the states with $L = 1, 2, 3$, respectively.
- (3) Large mass differences for high excitations like $D(2^3D_3)$ and $D(2^3F_4)$ reach 120 MeV and 180 MeV compared to the predictions in Ref. [19].
- (4) The recently observed $D(2760)$ and $D(2750)$ resonances are interpreted as the $D(1^3D_3)$ and the low-mass $D(1D_2^L)$ states, where $D(1D_2^L)$ is in fact the state with $j_q = l + 1/2$ (see Sec. V) and therefore should have relatively small total width, as is observed in the BABAR experiments [5]. It implies that in the LS scheme the states $D(1^3D_2)$ and $D(1^1D_2)$ are mixed with the mixing angle $\theta = 40^\circ$.

IV. HIGHER D_s MESONS

For the S -wave states, there are no string corrections; nevertheless, the mass $M(D_s(3^1S_0)) = 3140 \text{ MeV}$ calculated here is 79 MeV less than the one given in Ref. [19] (see Table IV). From our point of view, this happens

because of the large constituent mass $\tilde{m}_s = 500$ MeV taken in Ref. [19]. To illustrate this effect we have solved the spinless Salpeter equation with two different masses of the s quark, $\tilde{m}_s = 0.5$ GeV and $m_s = 0.2$ GeV, keeping all other parameters the same. Then the mass difference $\delta(nL) = M_{\text{cog}}(nL, m_1 = 0.5 \text{ GeV}) - M_{\text{cog}}(nL, m_1 = 0.2 \text{ GeV})$ appears to be almost constant for a fixed L and changing n : $\delta(2P) \simeq \delta(3P) = -138$ MeV, $\delta(1D) \simeq \delta(2D) = -130$ MeV, and $\delta(1F) \simeq \delta(2F) = -120$ MeV. Thus one may expect mass differences ~ 100 – 150 MeV to occur between relativistic models with large constituent light (strange) quark mass compared to the RSH, which uses current quark masses.

Just for that reason the masses $M(D_s(2^3D_3))$ and $M(D_s(1^3F_4))$ are in our calculations ~ 180 MeV lower than in Ref. [19] and again such a large difference cannot be explained by the string corrections, which is only ~ -45 MeV for the $D_s(2F)$ state.

Thus one can conclude that a large decrease in the masses of higher states predicted here mainly comes from two sources: the string correction and the use of the current mass for an s quark, which is significantly smaller than a typical constituent mass $\tilde{m}_s \sim 450 \pm 50$ MeV.

There exists another characteristic feature of the D_s spectrum—for all known states the experimental masses of $D_s(nL)$ and $D(nL)$ differ by ~ 100 MeV. In our calculations such a spacing $\delta_s(nL)$ comes from two sources: first, from different e.v. $M_0(nL)$ of Eq. (5) in the cases with $m_q = 0$ and $m_s = 0.20$ GeV, which give $\sim 50 \pm 10$ MeV difference; second, the light and the s quarks have different nonperturbative SE corrections (negative), which are $\sim 40 \pm 10$ MeV smaller for the s quark as compared to a light quark. Altogether δ_s appears to be ~ 100 MeV for low-lying states and smaller, $\delta_s \sim 70$ – 80 MeV, for higher states.

Our results about the D_s spectrum can be summarized as follows:

- (1) The HF splitting between $D_s(2^3S_1)$ and $D_s(2^1S_0)$, calculated with the use of the universal $\alpha_{\text{HF}} = 0.31$ from Ref. [27], gives good agreement with experiment.
- (2) The resonance $D_s(3044)$ is considered here as the high-mass state $D_s(2P_1^H)$, which is dominantly the state with $j_s = 1/2$ (see Sec. V) and therefore has to have large total width, in agreement with the experimental value $\Gamma = 239 \pm 35$ MeV; also its mass, 3040 MeV, calculated here, is in full agreement with experiment.
- (3) The resonance $D_s(2860)$ is interpreted as the $D_s(1^3D_3)$ state and its calculated mass 2840 MeV is in agreement with experiment. This state with $J^P = 3^-$ and $j_s = 5/2$ is assumed to have relatively small total width, as it takes place for $D_{s2}^*(2573)$. Indeed, the experimental width $\Gamma(D(2860)) = 48 \pm 3$ MeV [2,4] is small for so high a resonance.

- (4) The calculated masses of the higher states, like $D_s(2D)$ and $D_s(2F)$, are 120–200 MeV less than those from Ref. [19].

V. FINE-STRUCTURE SPLITTINGS

On a fundamental level, the spin-dependent potentials $V_i(r)$ ($i = 1$ – 4) have been studied in analytical approaches [39,42], and also on the lattice [43], where the spin-dependent potentials are expressed via the vacuum correlators. When the spin-orbit potential $V_{\text{SO}}(r)$ is considered, its perturbative part can be expressed only through the vector potential $V_2(r) \equiv V(r)$ if the Gromes relation [44] is used, and its nonperturbative part is determined by the scalar confining potential $S(r) = \sigma r$. For the tensor potential the nonperturbative contribution appears to be very small [39,43] and it is defined by the perturbative potential only, usually denoted by $V_3(r)$, which in general case is not equal to $[\frac{V}{r} - V'']$, as it takes place for the one-gluon-exchange (OGE) potential [notice that in the static potential (8) the effective vector coupling $\alpha_B(r)$ includes higher order perturbative corrections, while these corrections appear to be different for different spin-dependent potentials and in OGE approximation they are neglected]:

$$V_3(r) = 3T_0(r)\xi \equiv \frac{4\alpha_{\text{FS}}}{3r^3} \xi. \quad (10)$$

Here the factor $\xi(nL)$ is introduced to show the difference between $V_3(r)$ and $3T_0(r)$. In heavy quarkonia this factor ξ appears due to second order perturbative corrections, being $\xi \simeq 1.30 \pm 0.05$, both in charmonium and bottomonium [30]. However, the value of ξ remains unknown for heavy-light mesons and the difference between $V_3(r)$ and $3T_0(r)$ may be important for the FS analysis. In heavy quarkonia for the $1P$ states the spin-orbit $a_{\text{SO}}(1P)$ and the tensor $t(1P)$ m.e. can be extracted from the experimental masses since all of them are measured with great accuracy.

The study of the FS of the $D(nL)$ and $D_s(nL)$ multiplets is a more complicated task, because only a few masses are known from experiment, besides those for the $D(1P)$ and $D_s(1P)$ multiplets. Moreover, many states lie above open thresholds and may have mass shifts, which change the mass values as compared to those in single-channel approximation. Nevertheless, a general analysis of the FS in heavy-light mesons is very useful and allows us to understand better the FS dynamics and make definite conclusions about mixing angles for the states with $J = L$.

For a multiplet nL the FS is considered here in the basis j_q^2 , where the total angular momentum of a light (strange) quark j_q is diagonal [22]. (Below we use the notation $j_q \equiv j$.) This basis is especially convenient for the calculation of the mixing angle [denoted as $\phi(nl)$] between the states with $j = l + 1/2$ and $j = l - 1/2$, if $J = l$. Another scheme, S^2 , is also often used, and in this scheme the notation θ for a mixing angle is used here. The relation between θ and ϕ can be easily established, writing the

high-mass state (L_J^H) and low-mass state (L_J^L) with $J = L$ in both schemes. In the j^2 basis we write

$$\begin{aligned} |J^H\rangle &= \sin\phi \left| j = l + \frac{1}{2} \right\rangle + \cos\phi \left| j = l - \frac{1}{2} \right\rangle, \\ |J^L\rangle &= \cos\phi \left| j = l + \frac{1}{2} \right\rangle - \sin\phi \left| j = l - \frac{1}{2} \right\rangle, \end{aligned} \quad (11)$$

while in the S^2 scheme the same physical states are defined as in Ref. [12],

$$\begin{aligned} |L^H\rangle &= -\sin\theta |^1L_J\rangle + \cos\theta |^3L_J\rangle, \\ |L^L\rangle &= \cos\theta |^1L_J\rangle + \sin\theta |^3L_J\rangle. \end{aligned} \quad (12)$$

Then taking from Ref. [22] the relations

$$\begin{aligned} |J = l, j = l - 1/2\rangle &= \sqrt{\frac{l+1}{2l+1}} |J = l, S = 1\rangle \\ &\quad - \sqrt{\frac{l}{2l+1}} |J = l, S = 0\rangle, \\ |J = l, j = l + 1/2\rangle &= \sqrt{\frac{l}{2l+1}} |J = l, S = 1\rangle \\ &\quad + \sqrt{\frac{l+1}{2l+1}} |J = l, S = 0\rangle, \end{aligned} \quad (13)$$

and inserting them into Eq. (11), one obtains

$$\theta = -\phi + \arccos\sqrt{\frac{l+1}{2l+1}}. \quad (14)$$

For $L = 1, 2, 3$ it gives

$$\begin{aligned} \theta(L = 1) &= -\phi + 35.26^\circ, \\ \theta(L = 2) &= -\phi + 39.23^\circ, \\ \theta(L = 3) &= -\phi + 40.89^\circ. \end{aligned} \quad (15)$$

To determine ϕ one needs to know the m.e. of the spin-orbit and tensor potentials, which are written here in a more general form than in Ref. [22]:

$$V_{SO} = \lambda_1(r) 2\mathbf{l} \cdot \mathbf{s}_1 + \lambda_2(r) 2\mathbf{l} \cdot \mathbf{s}_2, \quad (16)$$

with

$$\begin{aligned} \lambda_1(r) &= \frac{1}{4\omega_1^2} \frac{V' - S'}{r} + \frac{1}{2\omega_1\omega_2} \frac{V'}{r}, \\ \lambda_2(r) &= \frac{1}{4\omega_2^2} \frac{V' - S'}{r} + \frac{1}{2\omega_1\omega_2} \frac{V'}{r}. \end{aligned} \quad (17)$$

Notice that the kinetic energies ω_i enter λ_i in Eq. (17) instead of the constituent masses usually used in potential models. This change follows from the general consideration of spin-dependent potentials in the RSH [39] and is important for higher states, decreasing their FS splittings.

For the linear confining potential $S' = \sigma$, while the perturbative vector potential $V(r)$ is taken in the form, satisfying the relation $V'(r)/r = 4\alpha_{FS}/r^3 \equiv T_0$, as for the OGE potential, where the vector coupling α_{FS} is considered as an effective coupling. Then the quantity V_3 in the tensor potential,

$$V_t(r) = \frac{V_3(r)}{12\omega_1\omega_2} S_{12}, \quad (18)$$

is given in Eq. (10) and the tensor operator is defined as usual by

$$S_{12} = 3 \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{r})(\boldsymbol{\sigma}_2 \cdot \mathbf{r})}{r^2} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2. \quad (19)$$

Later we use for simplicity the notations $\lambda_i(nl)$ for m.e. $\langle \lambda_i(r) \rangle_{nl}$ and

$$t(nl) = \left\langle \frac{V_3(r)}{3\omega_1\omega_2} \right\rangle_{nl}. \quad (20)$$

The spin-orbit m.e. a_{SO} , given by

$$a_{SO}(nl) = \lambda_1(nl) + \lambda_2(nl), \quad (21)$$

and the tensor m.e. $t(nl)$ fully determine the FS splittings for the states with $J = l + 1$ and $J = l - 1$ (in both cases spin $S = 1$):

$$\begin{aligned} M(J = l + 1, S = 1) &= M_{\text{cog}} + la_{SO} - \frac{l}{2(2l+3)} t, \\ M(J = l - 1, S = 1) &= M_{\text{cog}} - (l+1)a_{SO} - \frac{l+1}{2(2l-1)} t. \end{aligned} \quad (22)$$

For $J = l$ the states with $j = l + 1/2$ and $j = l - 1/2$ are mixed and their masses and mixing angles ϕ are defined by the matrix M_{mix} :

$$M_{\text{mix}} = \begin{pmatrix} a_{SO}l + \frac{l}{2(2l+1)}[t - 8\lambda_2(l+1)] & -(4\lambda_2 - t) \frac{\sqrt{l(l+1)}}{2(2l+1)} \\ -(4\lambda_2 - t) \frac{\sqrt{l(l+1)}}{2(2l+1)} & -a_{SO}(l+1) + \frac{l+1}{2(2l+1)}(t + 8l\lambda_2) \end{pmatrix}. \quad (23)$$

From Eq. (23) one can see that in general the matrix M_{mix} depends on a_{SO} and t , and also on the m.e. λ_2 , and the value of the factor $4\lambda_2 - t$, present in the off-diagonal m.e., is important for the determination of the mixing effect.

In the heavy-quark limit there is no mixing, because both λ_2 and t are going to zero [they are proportional to $m_Q^{-n}(n = 1, 2)$ and may therefore be neglected]. Then the high-mass state H has $j = l + 1/2$ while the low-mass state has $j = l - 1/2$, if a_{SO} is positive. However, such a situation with $a_{\text{SO}} \gg t$ does not occur even in bottomonium, where for the $1P$ states $a_{\text{SO}}(b\bar{b}, \text{exp}) = 13.65 \pm 0.39$ MeV coincides with $t(b\bar{b}, \text{exp}) = 13.13 \pm 1.04$ MeV within the experimental errors, and in charmonium $a_{\text{SO}}(c\bar{c}, \text{exp}) = 34.96 \pm 0.13$ MeV is even 14% smaller than $t(c\bar{c}, \text{exp}) = 40.63 \pm 0.26$ MeV (their ratio is 0.86).

Such a decrease of the spin-orbit m.e. and the ratio a_{SO}/t occurs due to the negative $-\sigma\langle r^{-1} \rangle_{nL}$ term and a partial or full cancellation in the m.e. $\langle \frac{V^l - S^l}{r} \rangle_{nL}$ is possible. (Also the m.e. $\langle r^{-3} \rangle_{nL}$, entering the spin-orbit and tensor m.e., decreases for increasing n and L .) Therefore it is of interest to define the quantity

$$A_{\text{SO}}(nL) = \frac{4}{3} \alpha_{\text{FS}} \langle r^{-3} \rangle_{nL} - \sigma \langle r^{-1} \rangle_{nL}, \quad (24)$$

which does not depend on ω_i and enters $\lambda_i = \frac{A_{\text{SO}}}{4\omega_i^2} + \frac{T_0}{2}$ ($i = 1, 2$). In the $D(D_s)$ mesons the factor $A_{\text{SO}}(nL)$ is negative and its magnitude depends on the value of α_{FS} taken. Here $\alpha_{\text{FS}} = 0.45$ is mostly used, which is a bit larger than $\alpha_{\text{SO}} \sim 0.38 \pm 0.02$ extracted from the charmonium FS [30]. The values of A_{SO} can be illustrated by the following numbers:

- (1) In bottomonium $A_{\text{SO}} \simeq 0.14$ GeV³ is positive and relatively large, while in charmonium $A_{\text{SO}} = \pm 0.01$ GeV³ is already small, even compatible with zero, so that the ratio $\frac{a_{\text{SO}}}{t} = 0.86$ is less than unity.
- (2) For the $D(nL)$ multiplets the factor A_{SO} is always negative: ~ -0.017 GeV³ for the $1P$, $2P$ states, and ~ -0.028 GeV³ for the $1D$ and $1F$ states (for $\alpha_{\text{FS}} = 0.45$). This result weakly depends on the quark masses used.
- (3) In M_{mix} a common scale is defined by the tensor m.e. t , and for $t = T_0$ (i.e., $\xi = 1.0$) it has values $t(1P) = 39$ MeV, $t(2P) = 29$ MeV, $t(1D) = 11.3$ MeV, $t(1F) = 5$ MeV.
- (4) For $L = 2, 3$ the mixing angle is very small, $|\phi| \leq 1^\circ$, for any reasonable choice of coupling. On the contrary, for the nP ($n = 1, 2$) states the mixing angle is very sensitive to α_{FS} used.
- (5) For small coupling, $\alpha_{\text{FS}} \leq 0.30$, the mixing angle ϕ decreases, so that the main uncertainty in any FS analysis comes from the value of α_{FS} taken, which is not fixed yet.

In our calculations the following values of the kinetic energies are obtained for $D(nL)$:

$$\begin{aligned} \omega_q(1P) &= 0.60 \text{ GeV}, & \omega_q(1D) &= 0.683 \text{ GeV}, \\ \omega_q(1F) &= 0.757 \text{ GeV}, & \omega_c(1P) &= 1.555 \text{ GeV}, \\ \omega_c(1D) &= 1.588 \text{ GeV}, & \omega_c(1F) &= 1.62 \text{ GeV}. \end{aligned} \quad (25)$$

For the D_s mesons the FS picture is essentially the same, because the m.e. for D_s , which are important for the FS, coincide within 1%–5% with those of the D mesons, and therefore the D_s FS splittings and mixing angles are practically the same as for the D mesons (see Tables II, III, and IV).

We also assume that for a given nL multiplet the masses of the $M(J = l + 1, j = l + 1/2)$ and $M(J = l, j = l + 1/2)$ states have no mass shifts (or have small mass shifts), as it happens for the $D(1P)$ and $D_s(1P)$ multiplets, and therefore the mass differences between these states,

$$\begin{aligned} M(D_{s2}^*(2573)) - M(D_{s1}(2535)) &= 37.31 \pm 1.0 \text{ MeV}, \\ M(D_2^*(2460)) - M(D_1(2422)) &= 40.8 \pm 1.6 \text{ MeV}, \end{aligned} \quad (26)$$

may be considered as the most stable characteristic of a given multiplet nL ; in general this mass difference is denoted by $\Delta(nl)$:

$$\begin{aligned} \Delta(j = l + 1/2) &= M(J = l + 1, j = l + 1/2) \\ &\quad - M(J = l, j = l + 1/2). \end{aligned} \quad (27)$$

Our calculations show that for the $D(1P)$ states the quantity $\lambda_1 = 5.5$ MeV is positive and small, while $\lambda_2 = 17.3$ MeV is relatively large, giving $a_{\text{SO}}(1P) = 22.8$ MeV, and $t(1P) = T_0(1P) = 38.6$ MeV ($\xi = 1.0$), so that the ratio $a_{\text{SO}}/t = 0.59$ is smaller than in charmonium, where this ratio is 0.86.

With the use of M_{mix} , Eq. (23), the mass splittings within the $2P$ multiplet are calculated (see Tables III and IV) and the mixing angle depends on α_{FS} , decreasing for smaller coupling: a large angle $\phi(1P) = -38^\circ$ is obtained for large $\alpha_{\text{FS}} = 0.60$, while $\phi(1P) = -12.6^\circ$ for $\alpha_{\text{FS}} = 0.45$ and $\phi = -4.2^\circ$ for a smaller $\alpha_{\text{FS}} = 0.33$.

For higher orbital excitations ($l = 2, 3, n = 1$) the non-diagonal terms in the matrix M_{mix} appear to be much smaller than the diagonal m.e. for $\alpha_{\text{FS}} = 0.45$ and due to this fact the mixing angles

$$\phi(1D) = -0.89^\circ, \quad \phi(1F) = -1.0^\circ \quad (28)$$

are very small. From Eq. (14) these values of ϕ correspond to the following angles θ between the states n^3L_J ($S = 1$) and n^1L_J ($S = 0$) with $J = L$: $\theta(1D) = 40^\circ$ and $\theta(1F) = 42^\circ$. This result may be important for the hadronic decays of these resonances [12].

The calculated mass differences $\Delta(nL)$, defined in Eq. (27),

$$\begin{aligned} \Delta(1P) &= 37.6 \text{ MeV}, & \Delta(1D) &= 15 \text{ MeV}, \\ \Delta(1F) &= 14 \text{ MeV}, \end{aligned} \quad (29)$$

are in good agreement with the experimental numbers: $\Delta(1P, \text{exp}) = 37.6 \text{ MeV}$ [1] and $\Delta(1D, \text{exp}) \approx M(D(2760)) - M(D(2750)) = (11 \pm 9) \text{ MeV}$ [4].

We do not discuss here the masses of the states with $j = l - 1/2$, which may have large mass shifts. In the single-channel approximation the mass $M(1P, 0^+)$ is 104 MeV smaller than $M(1P, 2^+)$; for $L = 2$ almost equal masses $M(1^3D_3)$ and $M(1^3D_1)$ are obtained, while for the $L = 3$ states $M(1^3F_4)$ is even smaller than $M(1^3F_2)$ (see Tables II and III). It does not exclude that because of possible mass shifts, the physical masses $M(1^3D_1)$ and $M(1^3F_2)$ become smaller than $M(1^3D_3)$ and $M(1^3F_4)$.

In Ref. [22] for the $1P$ states the approximation $\lambda_2 = \frac{t}{2}$ was used, which in our consideration is also valid for the $1P$ and $2P$ states. For the $1D$ and $1F$ states the factor λ_2 is smaller, $\lambda_2(1D) \sim 0.3t$, $\lambda_2(1F) \sim 0.2t$, and therefore the factor $4\lambda_2 - t$ in the off-diagonal term in Eq. (23) is also smaller. However, the main reason why a small mixing occurs for $l = 2, 3$ is that the diagonal terms appear to be larger than the off-diagonal terms due to larger algebraic coefficients.

As a result, for $l = 2$ and 3 the high-mass state is dominantly the state with $j = l - 1/2$ and the low-mass state is mostly the state with $j = l + 1/2$.

VI. CONCLUSIONS

The spectra of the D and D_s mesons were studied with the use of the RSH, where only such fundamental parameters as the string tension and the quark current masses

enter, and the only uncertainty comes from the freezing constant α_{crit} , which for higher states gives a theoretical error $\lesssim 20 \text{ MeV}$ in the spin-averaged mass. We have shown that

- (1) The calculated masses of the higher excitations appear to be 50–150 MeV lower than in other RQM with the same string tension $\sigma = 0.18 \text{ GeV}^2$. It occurs for two reasons: first, due to the string corrections for the states with $L \neq 0$ and, second, because we use the current quark masses.
- (2) Using the j^2 basis, the states with $J = l$ are shown to have very small mixing angles for $l = 2$ and 3 : $\phi \approx -1^\circ$. It means that the states 1^3L_L and 1^1L_L are mixed with $\theta(1D) = 40^\circ$ and $\theta(1F) = 42^\circ$.
- (3) The calculated masses of the state 1^3D_3 and the low-mass state $1D_2^L$ agree with the new BABAR resonances, $D(2760)$ and $D(2750)$.
- (4) The resonance $D_{sJ}^*(2860)$ is considered as the $1D_s(1^3D_3)$ state and $D_s(3040)$ as the high-mass $2P_1^H$ state.
- (5) For the yet unobserved resonances the following masses are predicted: $M(D(2^3P_2)) = 2965 \text{ MeV}$, $M(2P_1^L) = 2940 \text{ MeV}$, $M(D(1^3F_4)) = 3030 \text{ MeV}$, and $M(D_s(1^3F_4)) = 3110 \text{ MeV}$.

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